

Sharpening the empirical claims of generative syntax through formalization

Tim Hunter

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NASSLLI, June 2014

Part 1: Grammars and cognitive hypotheses

What is a grammar?

What can grammars do?

Concrete illustration of a target: Surprisal

Parts 2–4: Assembling the pieces

Minimalist Grammars (MGs)

MGs and MCFGs

Probabilities on MGs

Part 5: Learning and wrap-up

Something slightly different: Learning model

Recap and open questions

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Part 3

MGs and MCFGs

Where we're up to

We've seen:

- MGs with operations defined that manipulated trees
- that the structure that “really matters” (e.g. for recursion) can be boiled down to funny-looking “derivation trees” (with things like t , $\{-k\}$ at the non-leaf nodes)

Now:

- A way to think of how these derivation trees relate to surface strings (without going via trees)
- In some ways not totally necessary for the rest of the course, but helpful

Later:

- Adding probabilities to MGs: in a way that sort of works, and does some good stuff, but doesn't do everything we'd want
- Adding probabilities to MGs: in an even better way

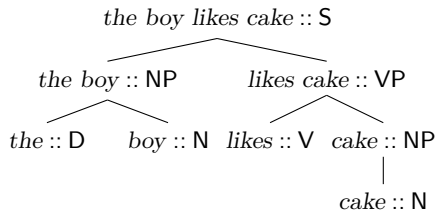
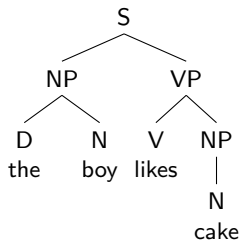
Outline

- 9 A different perspective on CFGs
- 10 Concatenative and non-concatenative operations
- 11 MCFGs
- 12 Back to MGs

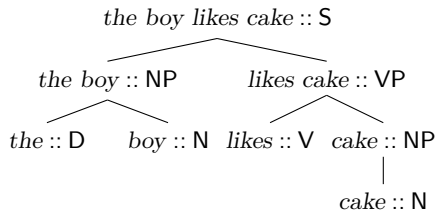
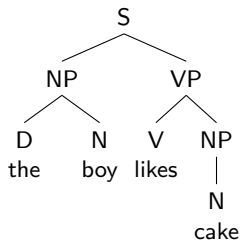
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Trees



Trees



How to think of a tree:

- less as a **picture of a string**
- more as a graphical representation of **how a string was constructed**, with the string “at” the top node

Two sides of a CFG rule

A rule like 'S \rightarrow NP VP' says two things:

- What combines with what:
An NP and a VP can combine to form an S
- How to produce a string of the new category:
Put the NP-string to the left of the VP-string

More explicitly:

$$st :: S \rightarrow s :: \text{NP} \quad t :: \text{VP}$$

Example: X-bar theory

Japanese

$XP \rightarrow \text{Spec } X'$

$X' \rightarrow \text{Comp } X$

English

$XP \rightarrow \text{Spec } X'$

$X' \rightarrow X \text{ Comp}$

Example: X-bar theory

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English

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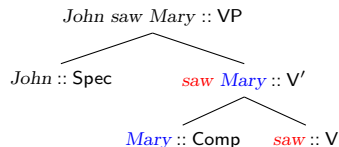
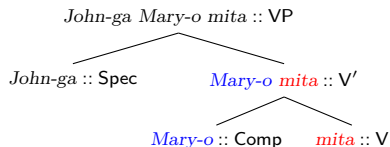
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English

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Concatenative and non-concatenative operations

Concatenative morphology:

play + ed \rightsquigarrow played

play + ing \rightsquigarrow playing

play + s \rightsquigarrow plays

Non-concatenative morphology:

(k,t,b) + (i,aa) \rightsquigarrow kitaab (“book”)

(k,t,b) + (aa,i) \rightsquigarrow kaatib (“writer”)

(k,t,b) + (ma,uu) \rightsquigarrow maktuub (“written”)

(k,t,b) + (a,i,a) \rightsquigarrow katiba (“document”)

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Concatenative syntax:

plays + tennis \rightsquigarrow plays tennis

plays + soccer \rightsquigarrow plays soccer

John + plays soccer \rightsquigarrow John plays soccer

Mary + plays soccer \rightsquigarrow Mary plays soccer

Concatenative and non-concatenative operations

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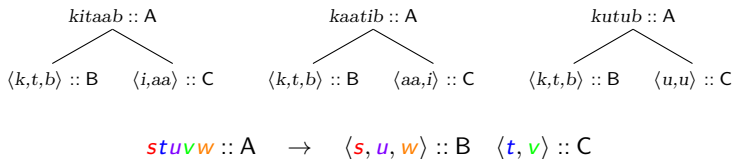
Concatenative syntax:

plays + tennis \rightsquigarrow plays tennis
 plays + soccer \rightsquigarrow plays soccer
 John + plays soccer \rightsquigarrow John plays soccer
 Mary + plays soccer \rightsquigarrow Mary plays soccer

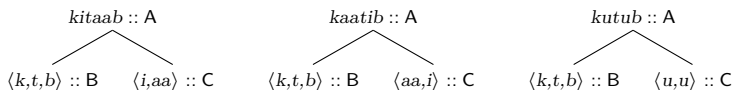
Non-concatenative syntax:

seems + (John, to be tall) \rightsquigarrow John seems to be tall
 seems + (Mary, to be intelligent) \rightsquigarrow Mary seems to be intelligent
 did + (John see, who) \rightsquigarrow who did John see
 did + (Mary meet, who) \rightsquigarrow who did Mary meet

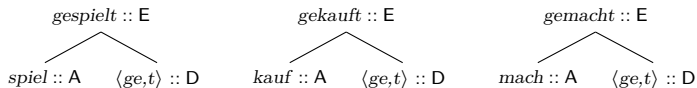
Non-concatenative morphology



Non-concatenative morphology



$stuvw :: A \rightarrow \langle s,u,w \rangle :: B \quad \langle t,v \rangle :: C$



$stu :: E \rightarrow t :: A \quad \langle s,u \rangle :: D$

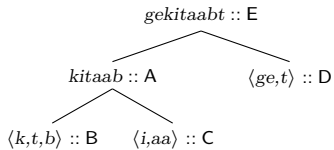
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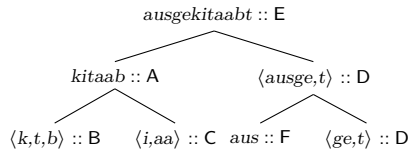
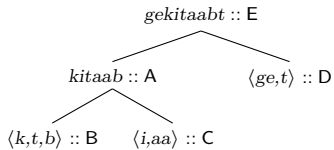


Non-concatenative morphology

$stuvw :: A \rightarrow \langle s, u, w \rangle :: B \quad \langle t, v \rangle :: C$

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$\langle ts, u \rangle :: D \rightarrow t :: F \quad \langle s, u \rangle :: D$

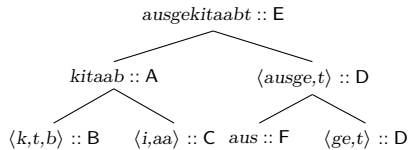
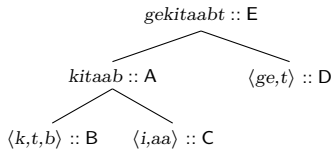


Non-concatenative morphology

$stuvw :: A \rightarrow \langle s, u, w \rangle :: B \quad \langle t, v \rangle :: C$

$stu :: E \rightarrow t :: A \quad \langle s, u \rangle :: D$

$\langle ts, u \rangle :: D \rightarrow t :: F \quad \langle s, u \rangle :: D$



If our goal is to characterize the array of well-formed/derivable objects — not to pronounce them — then all we care about is “what’s built out of what”:

$A \rightarrow B \ C$

$E \rightarrow A \ D$

$D \rightarrow F \ D$

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Multiple Context-Free Grammars (MCFGs)

$$st :: S \rightarrow s :: \text{NP} \quad t :: \text{VP}$$

An MCFG generalises to allow yields to be *tuples of strings*.

$$t_2 s t_1 :: Q \rightarrow s :: \text{NP} \quad \langle t_1, t_2 \rangle :: \text{VPWH}$$

This rule says two things:

- We can combine an NP with a VPWH to make a Q.
- The yield of the Q is $t_2 s t_1$, where s is the yield of the NP and $\langle t_1, t_2 \rangle$ is the yield of the VPWH.

Multiple Context-Free Grammars (MCFGs)

$$st :: S \rightarrow s :: NP \quad t :: VP$$

An MCFG generalises to allow yields to be *tuples of strings*.

$$t_2st_1 :: Q \rightarrow s :: NP \quad \langle t_1, t_2 \rangle :: VPWH$$

This rule says two things:

- We can combine an NP with a VPWH to make a Q.
- The yield of the Q is t_2st_1 ,
where s is the yield of the NP and $\langle t_1, t_2 \rangle$ is the yield of the VPWH.

$$\textit{which girl the boy says is tall} :: Q \rightarrow$$

$$\textit{the boy} :: NP \quad \langle \textit{says is tall}, \textit{which girl} \rangle :: VPWH$$

Some technical details

- Each nonterminal has a rank n , and yields only n -tuples of strings.

So given this rule:

$$t_2 s t_1 :: Q \rightarrow s :: NP \langle t_1, t_2 \rangle :: VPWH$$

we know that anything producing a VPWH must produce a 2-tuple.

$$\langle \dots, \dots \rangle :: VPWH \rightarrow \dots$$

and that anything producing an NP must produce a 1-tuple:

$$\dots :: NP \rightarrow \dots$$

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- The string-composition functions cannot copy pieces of their arguments.

OK	$s t :: VP$	\rightarrow	$s :: V$	$t :: NP$
OK	$t s \textit{ himself} :: S$	\rightarrow	$s :: V$	$t :: NP$
Not OK	$t s t :: S$	\rightarrow	$s :: V$	$t :: NP$

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- Each nonterminal has a rank n , and yields only n -tuples of strings.

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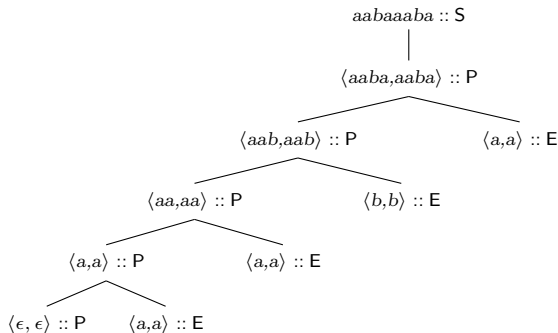
Not OK $t s t :: S \rightarrow s :: V \quad t :: NP$

- Essentially equivalent to [linear context-free rewriting systems](#) (LCFRSs).

Beyond context-free

$$\begin{aligned}
 t_1 t_2 :: S &\rightarrow \langle t_1, t_2 \rangle :: P \\
 \langle t_1 u_1, t_2 u_2 \rangle :: P &\rightarrow \langle t_1, t_2 \rangle :: P \quad \langle u_1, u_2 \rangle :: E \\
 \langle \epsilon, \epsilon \rangle &:: P \\
 \langle a, a \rangle &:: E \\
 \langle b, b \rangle &:: E
 \end{aligned}$$

$$\{ ww \mid w \in \{a, b\}^* \}$$



Unlike in a CFG, we can ensure that the two “halves” are extended in the same ways without concatenating them together.

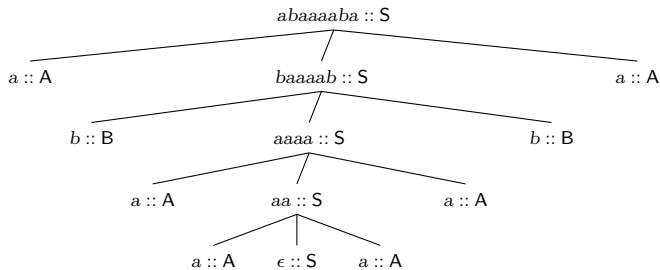
For comparison

$$t_1 s t_2 :: S \rightarrow t_1 :: A \quad s :: S \quad t_2 :: A$$

$$t_1 s t_2 :: S \rightarrow t_1 :: B \quad s :: S \quad t_2 :: B$$

$$\epsilon :: S$$

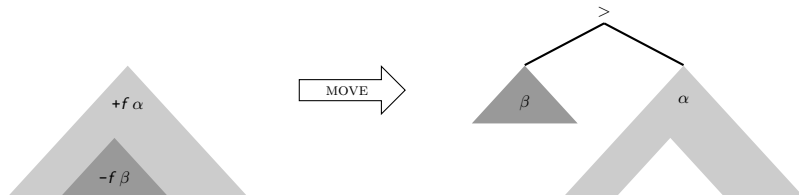
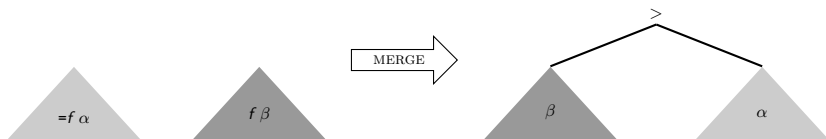
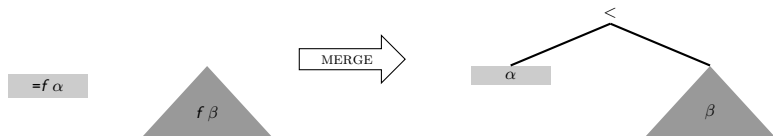
$$a :: A$$

$$b :: B$$


Outline

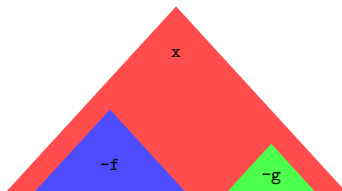
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Merge and move



What matters in a (derived) tree

This tree:



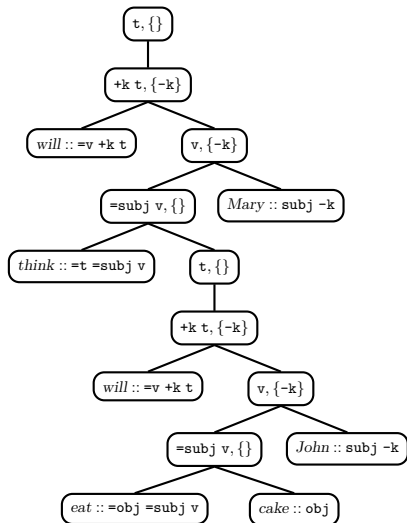
becomes a tuple of categorized strings:

$$\langle s :: x, t :: -f, u :: -g \rangle_0$$

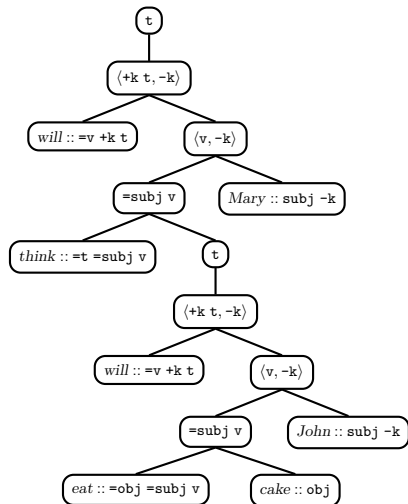
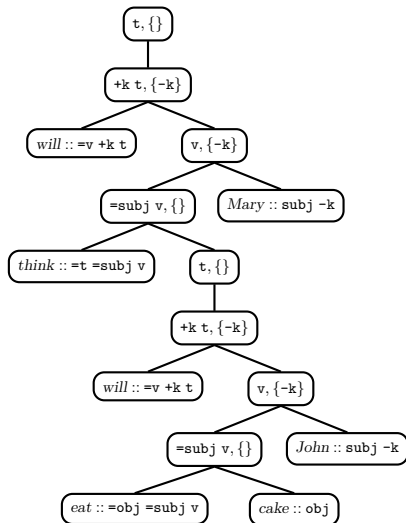
or, equivalently, a tuple-of-strings, categorized by a tuple-of-categories:

$$\langle s, t, u \rangle :: \langle x, -f, -g \rangle_0$$

Remember MG derivation trees?

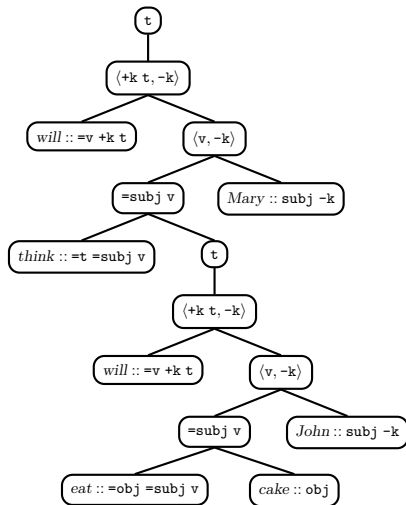


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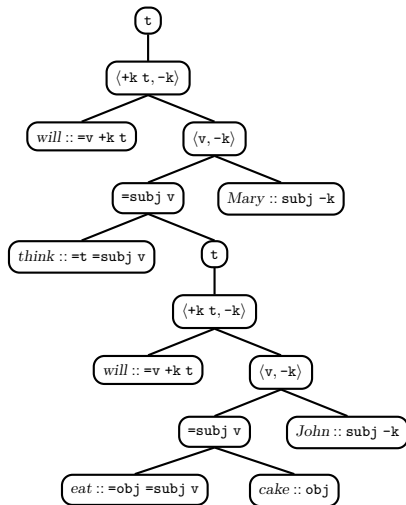
Slight change of notation (sorry):
internal node labels are now **lists of feature-lists**.



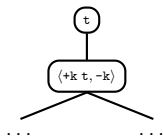
Remember MG derivation trees?

Slight change of notation (sorry):
internal node labels are now **lists of feature-lists**.

- We can tell that this tree represents a well-formed derivation, by checking the feature-manipulations at each step.
- How can we work out which string it derives?
 - Build up a tree according to merge and move rules, and read off leaves of the tree.
 - But there's a simpler way.



Producing a string from a derivation tree

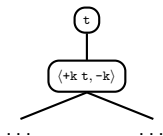


What do we need to have computed at the $\langle +k \ t, -k \rangle$ node, in order to compute the final string

Mary will think John will eat cake

at the t node?

Producing a string from a derivation tree

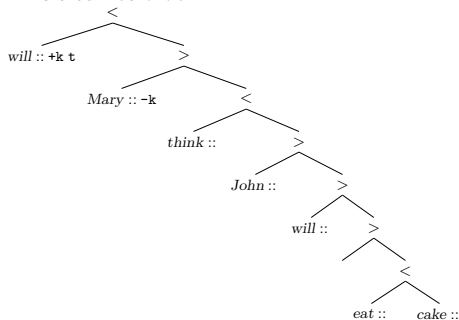


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This tree would do:

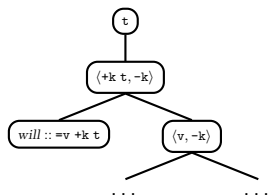


But all we actually need to know is:

- What's the string corresponding to the part that's going to move to check $-k$?
- What's the string corresponding to the leftovers?

These questions are answered by the tuple $\langle \textit{will think John will eat cake}, \textit{Mary} \rangle$

Producing a string from a derivation tree

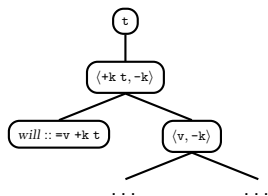


What do we need to have computed at the $\langle v, -k \rangle$ node, in order to compute the desired tuple

$\langle \textit{will think John will eat cake, Mary} \rangle$

at the $\langle +k t, -k \rangle$ node?

Producing a string from a derivation tree

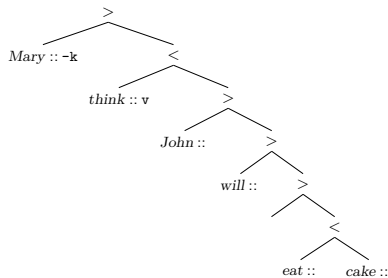


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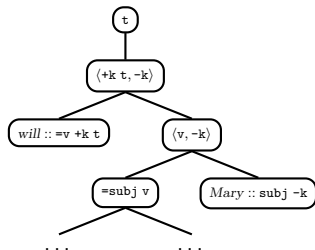


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Producing a string from a derivation tree

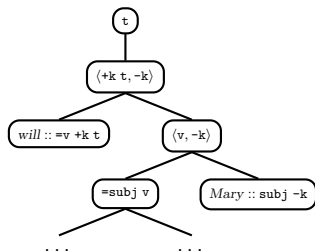


What do we need to have computed at the =subj v node, in order to compute the desired tuple

<think John will eat cake, Mary>

at the <v, -k> node?

Producing a string from a derivation tree

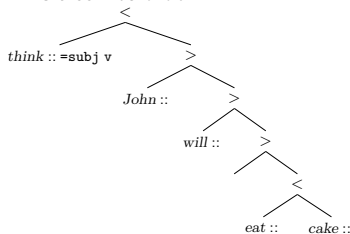


What do we need to have computed at the =subj v node, in order to compute the desired tuple

$\langle \text{think John will eat cake, Mary} \rangle$

at the $\langle v, -k \rangle$ node?

This tree would do:



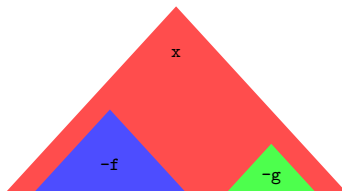
But all we actually need to know is:

- What's the string corresponding to the entire tree? (The "leftovers after no movement".)

This question is answered by the string *think John will eat cake*

What matters in a (derived) tree

This tree:



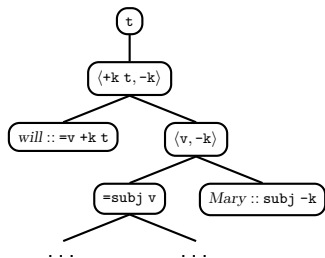
becomes a tuple of categorized strings:

$$\langle s :: x, t :: -f, u :: -g \rangle_0$$

or, equivalently, a tuple-of-strings, categorized by a tuple-of-categories:

$$\langle s, t, u \rangle :: \langle x, -f, -g \rangle_0$$

MCFG rules



$$t_2 t_1 :: t \rightarrow \langle t_1, t_2 \rangle :: \langle +k t, -k \rangle$$

$$\text{Mary will think John will eat cake} :: t \rightarrow \langle \text{will think John will eat cake, Mary} \rangle :: \langle +k t, -k \rangle$$

$$\langle s t_1, t_2 \rangle :: \langle +k t, -k \rangle \rightarrow s :: =v +k t \quad \langle t_1, t_2 \rangle :: \langle v, -k \rangle$$

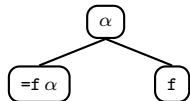
$$\langle \text{will think John will eat cake, Mary} \rangle :: \langle +k t, -k \rangle \rightarrow \text{will} :: =v +k t \quad \langle \text{think John will eat cake, Mary} \rangle :: \langle v, -k \rangle$$

$$\langle s, t \rangle :: \langle v, -k \rangle \rightarrow s :: =subj v \quad t :: subj -k$$

$$\langle \text{think John will eat cake, Mary} \rangle :: \langle v, -k \rangle \rightarrow \text{think John will eat cake} :: =subj v \quad \text{Mary} :: subj -k$$

One slightly annoying wrinkle

We know that this is a valid derivational step:



What is the corresponding MCFG rule?

Selected thing on the right?

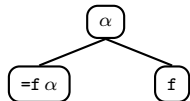
$$st :: \alpha \rightarrow s :: =f \alpha \quad t :: f$$

Selected thing on the left?

$$ts :: \alpha \rightarrow s :: =f \alpha \quad t :: f$$

One slightly annoying wrinkle

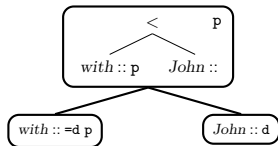
We know that this is a valid derivational step:



What is the corresponding MCFG rule?

Selected thing on the right?

$$st :: \alpha \rightarrow s :: =f \alpha \quad t :: f$$

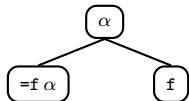


Selected thing on the left?

$$ts :: \alpha \rightarrow s :: =f \alpha \quad t :: f$$

One slightly annoying wrinkle

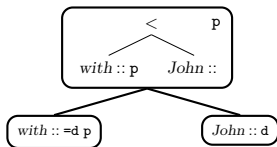
We know that this is a valid derivational step:



What is the corresponding MCFG rule?

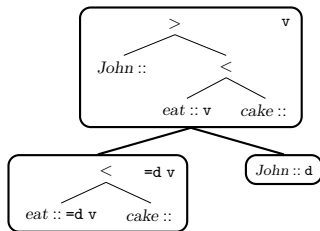
Selected thing on the right?

$$st :: \alpha \rightarrow s :: =f \alpha \quad t :: f$$



Selected thing on the left?

$$ts :: \alpha \rightarrow s :: =f \alpha \quad t :: f$$



One slightly annoying wrinkle

Each type needs to record not only the unchecked features, but also **whether the expression is lexical**.

I'll write lexical types as $\langle \dots \rangle_1$ and non-lexical types as $\langle \dots \rangle_0$.

So types of the form $\langle =f \alpha \rangle_1$ act slightly differently from those of the form $\langle =f \alpha \rangle_0$.

$$\begin{array}{l}
 st :: \langle \alpha \rangle_0 \quad \rightarrow \quad s :: \langle =f \alpha \rangle_1 \quad t :: \langle f \rangle_n \\
 \text{with John} :: \langle p \rangle_0 \quad \rightarrow \quad \text{with} :: \langle =d p \rangle_1 \quad \text{John} :: \langle d \rangle_1
 \end{array}$$

$$\begin{array}{l}
 ts :: \langle \alpha \rangle_0 \quad \rightarrow \quad s :: \langle =f \alpha \rangle_0 \quad t :: \langle f \rangle_n \\
 \text{John eat cake} :: \langle v \rangle_0 \quad \rightarrow \quad \text{eat cake} :: \langle =d v \rangle_0 \quad \text{John} :: \langle d \rangle_1
 \end{array}$$

Context-free structure

Schemas for MERGE steps:

$$\begin{aligned}\langle \gamma, \alpha_1, \dots, \alpha_j, \beta_1, \dots, \beta_k \rangle &\rightarrow \langle =\mathbf{f}\gamma, \alpha_1, \dots, \alpha_j \rangle \quad \langle \mathbf{f}, \beta_1, \dots, \beta_k \rangle \\ \langle \gamma, \alpha_1, \dots, \alpha_j, \delta, \beta_1, \dots, \beta_k \rangle &\rightarrow \langle =\mathbf{f}\gamma, \alpha_1, \dots, \alpha_j \rangle \quad \langle \mathbf{f}\delta, \beta_1, \dots, \beta_k \rangle\end{aligned}$$

Schemas for MOVE steps:

$$\begin{aligned}\langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_k \rangle &\rightarrow \langle +\mathbf{f}\gamma, \alpha_1, \dots, \alpha_{i-1}, -\mathbf{f}, \alpha_{i+1}, \dots, \alpha_k \rangle \\ \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \delta, \alpha_{i+1}, \dots, \alpha_k \rangle &\rightarrow \langle +\mathbf{f}\gamma, \alpha_1, \dots, \alpha_{i-1}, -\mathbf{f}\delta, \alpha_{i+1}, \dots, \alpha_k \rangle\end{aligned}$$

Context-free structure

Schemas for MERGE steps:

$$\begin{aligned} \langle \gamma, \alpha_1, \dots, \alpha_j, \beta_1, \dots, \beta_k \rangle &\rightarrow \langle =\mathbf{f}\gamma, \alpha_1, \dots, \alpha_j \rangle \quad \langle \mathbf{f}, \beta_1, \dots, \beta_k \rangle \\ \langle \gamma, \alpha_1, \dots, \alpha_j, \delta, \beta_1, \dots, \beta_k \rangle &\rightarrow \langle =\mathbf{f}\gamma, \alpha_1, \dots, \alpha_j \rangle \quad \langle \mathbf{f}\delta, \beta_1, \dots, \beta_k \rangle \end{aligned}$$

Schemas for MOVE steps:

$$\begin{aligned} \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_k \rangle &\rightarrow \langle +\mathbf{f}\gamma, \alpha_1, \dots, \alpha_{i-1}, -\mathbf{f}, \alpha_{i+1}, \dots, \alpha_k \rangle \\ \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \delta, \alpha_{i+1}, \dots, \alpha_k \rangle &\rightarrow \langle +\mathbf{f}\gamma, \alpha_1, \dots, \alpha_{i-1}, -\mathbf{f}\delta, \alpha_{i+1}, \dots, \alpha_k \rangle \end{aligned}$$

- MOVE steps **change** something without **combining** it with anything
- Compare with unary CFG rules, or type-raising in CCG, or ...

Three schemas for MERGE rules:

$$\langle st, t_1, \dots, t_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_k \rangle_0 \rightarrow \\ s :: \langle =f\gamma \rangle_1 \quad \langle t, t_1, \dots, t_k \rangle :: \langle f, \alpha_1, \dots, \alpha_k \rangle_n$$

$$\langle ts, s_1, \dots, s_j, t_1, \dots, t_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_j, \beta_1, \dots, \beta_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_j \rangle :: \langle =f\gamma, \alpha_1, \dots, \alpha_j \rangle_0 \quad \langle t, t_1, \dots, t_k \rangle :: \langle f, \beta_1, \dots, \beta_k \rangle_n$$

$$\langle s, s_1, \dots, s_j, t, t_1, \dots, t_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_j, \delta, \beta_1, \dots, \beta_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_j \rangle :: \langle =f\gamma, \alpha_1, \dots, \alpha_j \rangle_n \quad \langle t, t_1, \dots, t_k \rangle :: \langle f\delta, \beta_1, \dots, \beta_k \rangle_{n'}$$

Two schemas for MERGE rules:

$$\langle s_i s, s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f, \alpha_{i+1}, \dots, \alpha_k \rangle_0$$

$$\langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \delta, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f\delta, \alpha_{i+1}, \dots, \alpha_k \rangle_0$$

- Billot, S. and Lang, B. (1989). The structure of shared forests in ambiguous parsing. In *Proceedings of the 1989 Meeting of the Association of Computational Linguistics*.
- Chomsky, N. (1965). *Aspects of the Theory of Syntax*. MIT Press, Cambridge, MA.
- Chomsky, N. (1980). *Rules and Representations*. Columbia University Press, New York.
- Ferreira, F. (2005). Psycholinguistics, formal grammars, and cognitive science. *The Linguistic Review*, 22:365–380.
- Gärtner, H.-M. and Michaelis, J. (2010). On the Treatment of Multiple-Wh Interrogatives in Minimalist Grammars. In Hanneforth, T. and Fanselow, G., editors, *Language and Logos*, pages 339–366. Akademie Verlag, Berlin.
- Gibson, E. and Wexler, K. (1994). Triggers. *Linguistic Inquiry*, 25:407–454.
- Hale, J. (2006). Uncertainty about the rest of the sentence. *Cognitive Science*, 30:643–672.
- Hale, J. T. (2001). A probabilistic early parser as a psycholinguistic model. In *Proceedings of the Second Meeting of the North American Chapter of the Association for Computational Linguistics*.
- Hunter, T. (2011). Insertion Minimalist Grammars: Eliminating redundancies between merge and move. In Kanazawa, M., Kornai, A., Kracht, M., and Seki, H., editors, *The Mathematics of Language (MOL 12 Proceedings)*, volume 6878 of *LNCS*, pages 90–107, Berlin Heidelberg. Springer.
- Hunter, T. and Dyer, C. (2013). Distributions on minimalist grammar derivations. In *Proceedings of the 13th Meeting on the Mathematics of Language*.
- Koopman, H. and Szabolcsi, A. (2000). *Verbal Complexes*. MIT Press, Cambridge, MA.

- Lang, B. (1988). Parsing incomplete sentences. In *Proceedings of the 12th International Conference on Computational Linguistics*, pages 365–371.
- Levy, R. (2008). Expectation-based syntactic comprehension. *Cognition*, 106(3):1126–1177.
- Michaelis, J. (2001). Derivational minimalism is mildly context-sensitive. In Moortgat, M., editor, *Logical Aspects of Computational Linguistics*, volume 2014 of *LNCS*, pages 179–198. Springer, Berlin Heidelberg.
- Miller, G. A. and Chomsky, N. (1963). Finitary models of language users. In Luce, R. D., Bush, R. R., and Galanter, E., editors, *Handbook of Mathematical Psychology*, volume 2. Wiley and Sons, New York.
- Morrill, G. (1994). *Type Logical Grammar: Categorical Logic of Signs*. Kluwer, Dordrecht.
- Nederhof, M. J. and Satta, G. (2008). Computing partition functions of pcfgs. *Research on Language and Computation*, 6(2):139–162.
- Seki, H., Matsumara, T., Fujii, M., and Kasami, T. (1991). On multiple context-free grammars. *Theoretical Computer Science*, 88:191–229.
- Stabler, E. P. (2006). Sideways without copying. In Wintner, S., editor, *Proceedings of The 11th Conference on Formal Grammar*, pages 157–170, Stanford, CA. CSLI Publications.
- Stabler, E. P. (2011). Computational perspectives on minimalism. In Boeckx, C., editor, *The Oxford Handbook of Linguistic Minimalism*. Oxford University Press, Oxford.
- Stabler, E. P. and Keenan, E. L. (2003). Structural similarity within and among languages. *Theoretical Computer Science*, 293:345–363.

- Vijay-Shanker, K., Weir, D. J., and Joshi, A. K. (1987). Characterizing structural descriptions produced by various grammatical formalisms. In *Proceedings of the 25th Meeting of the Association for Computational Linguistics*, pages 104–111.
- Weir, D. (1988). *Characterizing mildly context-sensitive grammar formalisms*. PhD thesis, University of Pennsylvania.
- Yngve, V. H. (1960). A model and an hypothesis for language structure. In *Proceedings of the American Philosophical Society*, volume 104, pages 444–466.