# Sharpening the empirical claims of generative syntax through formalization

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ESSLLI, August 2015

Part 1: Grammars and cognitive hypotheses

What is a grammar?

What can grammars do?
Concrete illustration of a target: Surprisal

Parts 2–4: Assembling the pieces

Minimalist Grammars (MGs)
MGs and MCEGs

Probabilities on MGs

Part 5: Learning and wrap-up

Something slightly different: Learning model Recap and open questions

# Sharpening the empirical claims of generative syntax through formalization

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Part 4

Probabilities on MG Derivations

#### Outline

- Easy probabilities with context-free structure
- Different frameworks
- 15 Problem #1 with the naive parametrization
- 16 Problem #2 with the naive parametrization
- 17 Solution: Faithfulness to MG operations

Easy probabilities

# Easy probabilities with context-free structure

#### Probabilistic CFGs

Easy probabilities

"What are the probabilities of the derivations?"

"What are the values of  $\lambda_1$ ,  $\lambda_2$ , etc.?"

 $S \to NP VP$  $\lambda_1$ 

 $\mathsf{NP} \to \mathsf{John}$ 

 $NP \rightarrow Mary$ 

 $\mathsf{VP} \to \mathsf{ran}$ 

 $\lambda_5$  $\mathsf{VP} \to \mathsf{V} \; \mathsf{NP}$ 

 $\lambda_6$  $VP \rightarrow VS$  $V \rightarrow believed$ 

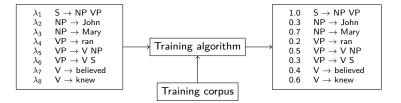
 $\lambda_7$  $V \rightarrow knew$ 

#### Probabilistic CFGs

Easy probabilities

"What are the probabilities of the derivations?"

"What are the values of  $\lambda_1$ ,  $\lambda_2$ , etc.?"



$$\lambda_5 = \frac{\mathsf{count}(\mathsf{VP} \to \mathsf{V} \; \mathsf{NP})}{\mathsf{count}(\mathsf{VP})}$$

#### MCFG for an entire Minimalist Grammar

#### Lexical items:

```
\epsilon :: \langle =t + wh c \rangle_1
                                                                                                  praise :: \langle =d v \rangle_1
        \epsilon :: \langle =t c \rangle_1
                                                                                                  marie :: (d)1
   will :: \langle =v = d t \rangle_1
                                                                                                  pierre :: \langle d \rangle_1
often :: \langle =v \ v \rangle_1
                                                                                                    who :: \langle d - wh \rangle_1
```

#### Production rules:

```
\langle st, u \rangle :: \langle +wh c, -wh \rangle_0 \rightarrow s :: \langle =t +wh c \rangle_1 \langle t, u \rangle :: \langle t, -wh \rangle_0
                         st :: \langle =d t \rangle_0 \rightarrow s :: \langle =v =d t \rangle_1 \quad t :: \langle v \rangle_0
   \langle st, u \rangle :: \langle =dt, -wh \rangle_0 \rightarrow s :: \langle =v =dt \rangle_1 \langle t, u \rangle :: \langle v, -wh \rangle_0
                                ts :: \langle c \rangle_0 \rightarrow \langle s, t \rangle :: \langle +wh c, -wh \rangle_0
                                st :: \langle c \rangle_0 \rightarrow s :: \langle =t c \rangle_1 \quad t :: \langle t \rangle_0
                                ts :: \langle \mathsf{t} \rangle_0 \rightarrow s :: \langle \mathsf{=d} \; \mathsf{t} \rangle_0 \quad t :: \langle \mathsf{d} \rangle_1
          \langle ts, u \rangle :: \langle t, -wh \rangle_0 \rightarrow \langle s, u \rangle :: \langle -dt, -wh \rangle_0 \quad t :: \langle d \rangle_1
                                st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1 \quad t :: \langle d \rangle_1
                                st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1 \quad t :: \langle v \rangle_0
             \langle s, t \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -d v \rangle_1 \quad t :: \langle d -wh \rangle_1
           \langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v, v \rangle_1 \langle t, u \rangle :: \langle v, -wh \rangle_0
```

Easy probabilities

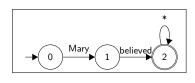
The context-free "backbone" for MG derivations identifies a parametrization for probability distributions over them.

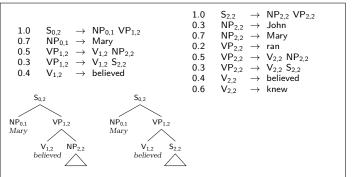
$$\lambda_2 = \frac{\mathsf{count}\big(\langle \mathtt{c} \rangle_0 \to \langle \mathtt{=t} \ \mathtt{c} \rangle_1 \langle \mathtt{t} \rangle_0\big)}{\mathsf{count}\big(\langle \mathtt{c} \rangle_0\big)}$$

Plus: It turns out that the intersect-with-an-FSA trick we used for CFGs also works for MCFGs!

# Grammar intersection example (simple)



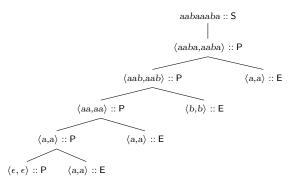




NB: Total weight in this grammar is not one! (What is it? Start symbol is  $S_{0.2}$ .) Each derivation has the weight "it" had in the original grammar.

# Beyond context-free

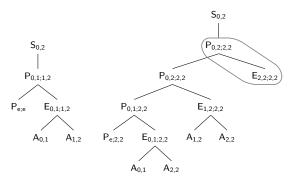
$$\begin{array}{lll} & & t_1t_2 :: \mathsf{S} & \to & \langle t_1, t_2 \rangle :: \mathsf{P} \\ \langle t_1u_1, t_2u_2 \rangle :: \mathsf{P} & \to & \langle t_1, t_2 \rangle :: \mathsf{P} & \langle u_1, u_2 \rangle :: \mathsf{E} \\ & \langle \epsilon, \epsilon \rangle :: \mathsf{P} & & & & & & & & \\ \langle a, a \rangle :: \mathsf{E} & & & & & & & & \\ \langle b, b \rangle :: \mathsf{E} & & & & & & & & & \end{array}$$



Unlike in a CFG, we can ensure that the two "halves" are extended in the same ways without concatenating them together.

Easy probabilities

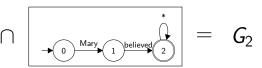
```
(b,b) :: E_{2,2;2,2}
                                                                                                                                                                \langle a,a\rangle :: \mathsf{E}_{2,2:2,2}
                                                                      S_{0,2}
                                                                                         \rightarrow \ P_{0,2;2,2}
                                                                                                                                                               \langle \epsilon, \epsilon \rangle :: P_{e:e}
                                                                      P_{0,2;2,2} \rightarrow P_{0,2;2,2} E_{2,2;2,2}
S<sub>0.2</sub>
                  \rightarrow \ P_{0,1;1,2}
                                                                                                                                                               \langle \epsilon, \epsilon \rangle :: P_{e;2,2}
                                                                      P_{0,2;2,2} \rightarrow P_{0,1;2,2} E_{1,2;2,2}
                            P_{e;e} \ E_{0,1;1,2}
P_{0,1;1,2} \rightarrow
                                                                                                                                                               a :: A_{2,2}
                                                                      P_{0,1;2,2} \rightarrow P_{e;2,2} E_{0,1;2,2}
                                                                                                                                                               b :: B_{2,2}
E_{0,1;1,2}
                            A_{0.1} A_{1.2}
                                                                      E_{0,1;2,2}
                                                                                         \rightarrow A<sub>0.1</sub> A<sub>2.2</sub>
                                                                      E_{1.2:2.2}
                                                                                         \rightarrow A<sub>1.2</sub> A<sub>2.2</sub>
                                                                                                                                                               a :: A_{0.1}
                                                                                                                                                              a :: A_{1,2}
```



## Intersection grammars

1.0  $S \rightarrow NP VP$ 0.3  $\mathsf{NP} \to \mathsf{John}$  $NP \rightarrow Mary$ 0.2  $VP \rightarrow ran$ 0.5  $VP \rightarrow V NP$ 0.3  $VP \rightarrow VS$ 0.4  $V \rightarrow believed$ 0.6  $V \to knew$ 

Easy probabilities



1.0  $S \rightarrow NP VP$ 0.3  $NP \rightarrow John$ 0.7  $NP \rightarrow Mary$  $VP \rightarrow ran$ 0.5  $VP \rightarrow V NP$ 0.3  $VP \rightarrow VS$ 0.4  $V \rightarrow believed$ 0.6  $V \rightarrow knew$ 



surprisal at 'John' 
$$= -\log P(W_3 = \text{John} \mid W_1 = \text{Mary}, W_2 = \text{believed})$$

$$= -\log \frac{\text{total weight in } G_3}{\text{total weight in } G_2}$$

$$= -\log \frac{0.0672}{0.224}$$

$$= 1.74$$

## Surprisal and entropy reduction

Easy probabilities

surprisal at 'John' 
$$= -\log P(W_3 = \text{John} \mid W_1 = \text{Mary}, W_2 = \text{believed})$$
  $= -\log \frac{\text{total weight in } G_3}{\text{total weight in } G_2}$ 

entropy reduction at 'John' = (entropy of  $G_2$ ) – (entropy of  $G_3$ )

## Computing sum of weights in a grammar ("partition function")

$$Z(A) = \sum_{A \to \alpha} \left( p(A \to \alpha) \cdot Z(\alpha) \right)$$

$$Z(\epsilon) = 1$$

$$Z(a\beta) = Z(\beta)$$

$$Z(B\beta) = Z(B) \cdot Z(\beta)$$
 where  $\beta \neq \epsilon$ 

(Nederhof and Satta 2008)

```
Z(V) = 0.4 + 0.6 = 1.0
       S \rightarrow NP VP
1.0
                                  Z(NP) = 0.3 + 0.7 = 1.0
0.3
       NP \rightarrow John
0.7
       NP \rightarrow Marv
                                  Z(VP) = 0.2 + (0.5 \cdot Z(V) \cdot Z(NP))
     VP \rightarrow ran
0.2
                                           = 0.2 + (0.5 \cdot 1.0 \cdot 1.0) = 0.7
       VP \rightarrow V NP
0.5
0.4
     V → believed
                                    Z(S) = 1.0 \cdot Z(NP) \cdot Z(VP)
0.6
       V \rightarrow knew
                                           = 0.7
```

```
S \rightarrow NP VP
1.0
0.3
        NP \rightarrow John
                                        Z(V) = 0.4 + 0.6 = 1.0
0.7
        NP \rightarrow Marv
                                      Z(NP) = 0.3 + 0.7 = 1.0
0.2
        VP \rightarrow ran
0.5
      VP \rightarrow V NP
                                      Z(VP) = 0.2 + (0.5 \cdot Z(V) \cdot Z(NP)) + (0.3 \cdot Z(V) \cdot Z(S))
0.3 \text{ VP} \rightarrow \text{V S}
                                        Z(S) = 1.0 \cdot Z(NP) \cdot Z(VP)
0.4
        V \rightarrow believed
0.6
        V \rightarrow knew
```

## Computing entropy of a grammar

Easy probabilities

```
h(S) = 0
1 0
       S \rightarrow NP VP
                              h(NP) = \text{entropy of } (0.3, 0.7)
0.3
       NP \rightarrow John
                              h(VP) = \text{entropy of } (0.2, 0.5, 0.3)
0.7
       NP \rightarrow Mary
                               h(V) = \text{entropy of } (0.4, 0.6)
      VP \rightarrow ran
0.2
0.5
      VP \rightarrow V NP
0.3
      VP \rightarrow VS
0.4
      V \rightarrow believed
                                H(S) = h(S) + 1.0(H(NP) + H(VP))
0.6
       V \rightarrow knew
                              H(NP) = h(NP)
                              H(VP) = h(VP) + 0.2(0) + 0.5(H(V) + H(NP)) + 0.3(H(V) + H(S))
                               H(V) = h(V)
```

## Surprisal and entropy reduction

Easy probabilities

surprisal at 'John' 
$$= -\log P(W_3 = \text{John} \mid W_1 = \text{Mary}, W_2 = \text{believed})$$
  
 $= -\log \frac{\text{total weight in } G_3}{\text{total weight in } G_2}$ 

entropy reduction at 'John' = (entropy of  $G_2$ ) – (entropy of  $G_3$ )

## Putting it all together (Hale 2006)

Easy probabilities

We can now put entropy reduction/surprisal together with a minimalist grammar to produce predictions about sentence comprehension difficulty!

complexity metric 
$$+$$
 grammar  $\longrightarrow$  prediction

- Write an MG that generates sentence types of interest
- Convert MG to an MCFG
- Add probabilities to MCFG based on corpus frequencies (or whatever else)
- Compute intersection grammars for each point in a sentence
- Calculate reduction in entropy across the course of the sentence (i.e. workload)

Demo

Easy probabilities

## Hale (2006)

Easy probabilities

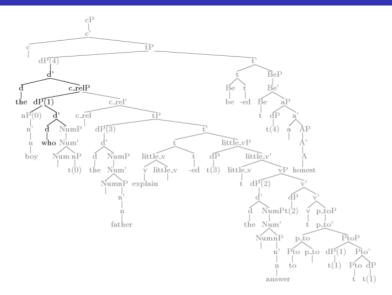


Fig. 11. Kaynian promotion analysis.

### Hale (2006)

they have -ed forget -en that the boy who tell -ed the story be -s so young the fact that the girl who pay -ed for the ticket be -s very poor doesnt matter I know that the girl who get -ed the right answer be -s clever he remember -ed that the man who sell -ed the house leave -ed the town

they have -ed forget -en that the letter which Dick write -ed yesterday be -s long the fact that the cat which David show -ed to the man like -s eggs be -s strange I know that the dog which Penny buy -ed today be -s very gentle he remember -ed that the sweet which David give -ed Sally be -ed a treat

they have -ed forget -en that the man who Ann give -ed the present to be -ed old the fact that the boy who Paul sell -ed the book to hate -s reading be -s strange I know that the man who Stephen explain -ed the accident to be -s kind he remember -ed that the dog which Mary teach -ed the trick to be -s clever

they have -ed forget -en that the box which Pat bring -ed the apple in be -ed lost the fact that the girl who Sue write -ed the story with be -s proud doesnt matter I know that the ship which my uncle take -ed Joe on be -ed interesting he remember -ed that the food which Chris pay -ed the bill for be -ed cheap

they have -ed forget -en that the girl whose friend buy -ed the cake be -ed wait -ing the fact that the boy whose brother tell -s lies be -s always honest surprise -ed us I know that the boy whose father sell -ed the dog be -ed very sad he remember -ed that the girl whose mother send -ed the clothe come -ed too late

they have -ed forget -en that the man whose house Patrick buy -ed be -ed so ill the fact that the sailor whose ship Jim take -ed have -ed one leg be -s important I know that the woman whose car Jenny sell -ed be -ed very angry he remember -ed that the girl whose picture Clare show -ed us be -ed pretty

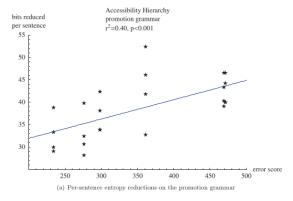
Easy probabilities

count	grammatical relation	definition
1430	subject	co-indexed trace is the first daughter of S
929	direct object	co-indexed trace is immediately following sister of a V-node
167	indirect object	co-indexed trace is part of a PP not annotated as benefactive, loca-
		tive, manner, purpose, temporal or directional
41	oblique	co-indexed trace is part of a benefactive, locative, manner, purpose,
		temporal or directional PP
34	genitive subject	WH word is whose and co-indexed trace is first daughter of S
4	genitive direct object	WH word is whose and co-indexed trace is immediately following
		sister of a V-node

Fig. 13. Counts from Brown portion of Penn Treebank III.

## Hale (2006)

Easy probabilities



```
Grammatical Relation:
                           SU
                                 DO
                                       IO
                                            OBL
                                                   GenS
                                                          GenO
     Repetition Accuracy:
                           406
                                 364
                                      342
                                             279
                                                    167
                                                           171
errors (= R.A._{max} - R.A.)
                           234
                                 276
                                      298
                                             361
                                                    471
                                                           469
```

Fig. 8. Results from Keenan and Hawkins (1987).

Easy probabilities

#### Hale actually wrote two different MGs:

- classical adjunction analysis of relative clauses
- Kaynian/promotion analysis

Easy probabilities

#### Hale actually wrote two different MGs:

- classical adjunction analysis of relative clauses
- Kaynian/promotion analysis

The branching structure of the two MCFGs was different enough to produce distinct Entropy Reduction predictions. (Same corpus counts!)

The Kaynian/promotion analysis produced a better fit for the Accessibility Hierarchy facts.

(i.e. holding the complexity metric fixed to argue for a grammar)

But there are some ways in which this method is insensitive to fine details of the MG formalism.

#### Outline

- 13 Easy probabilities with context-free structur
- Different frameworks
- 15 Problem #1 with the naive parametrization
- Problem #2 with the naive parametrization
- Solution: Faithfulness to MG operations

## Subtlely different minimalist frameworks

Minimalist grammars with many choices of different bells and whistles can all be expressed with context-free derivational structure.

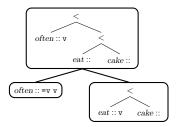
- Must keep an eye on finiteness of number of types (SMC or equivalent)!
- See Stabler (2011)

#### Some points of variation:

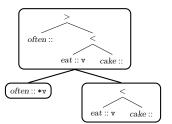
- adjunction
- head movement
- phases
- move as re-merge
- . . .

## How to deal with adjuncts?

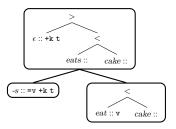
#### A normal application of MERGE?



#### Or a new kind of feature and distinct operation ADJOIN?

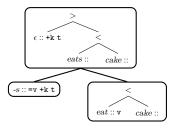


Modify  $_{\rm MERGE}$  to allow some additional string-shuffling in head-complement relationships?

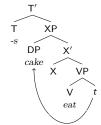


## How to implement "head movement"?

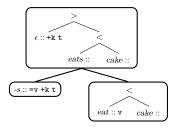
Modify MERGE to allow some additional string-shuffling in head-complement relationships?



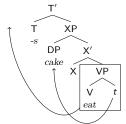
Or some combination of normal phrasal movements? (Koopman and Szabolcsi 2000)



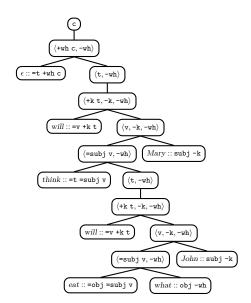
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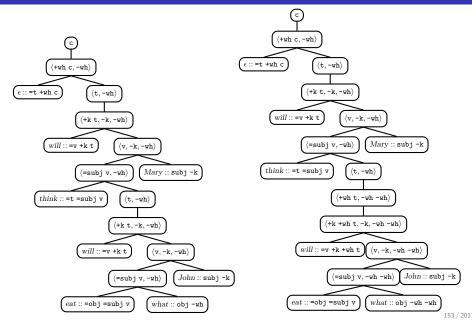
Or some combination of normal phrasal movements? (Koopman and Szabolcsi 2000)



# Successive cyclic movement?



## Successive cyclic movement?



## Unifying feature-checking (one way)

```
( John will seem to eat cake )

MOVE

(will seem to eat cake , John )

MERGE

(will ) (seem to eat cake , John )
```

```
(John will seem to eat cake)

MRG

(will seem to eat cake, John)

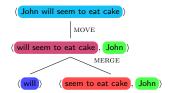
MRG

will, seem to eat cake, John)

INSERT

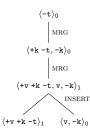
(will) (will seem to eat cake, John)
```

## Unifying feature-checking (one way)









$$\langle \mathsf{st}, \mathsf{t}_1, \dots, \mathsf{t}_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_k \rangle_0 \rightarrow \\ \mathsf{s} :: \langle = \mathsf{f} \gamma \rangle_1 \quad \langle \mathsf{t}, \mathsf{t}_1, \dots, \mathsf{t}_k \rangle :: \langle \mathsf{f}, \alpha_1, \dots, \alpha_k \rangle_n$$

$$\langle ts, s_1, \ldots, s_j, t_1, \ldots, t_k \rangle :: \langle \gamma, \alpha_1, \ldots, \alpha_j, \beta_1, \ldots, \beta_k \rangle_0 \rightarrow \\ \langle s, s_1, \ldots, s_j \rangle :: \langle = f\gamma, \alpha_1, \ldots, \alpha_j \rangle_0 \quad \langle t, t_1, \ldots, t_k \rangle :: \langle f, \beta_1, \ldots, \beta_k \rangle_n$$

$$\langle s, s_1, \dots, s_j, t, t_1, \dots, t_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_j, \delta, \beta_1, \dots, \beta_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_j \rangle :: \langle = f\gamma, \alpha_1, \dots, \alpha_j \rangle_n \quad \langle t, t_1, \dots, t_k \rangle :: \langle f\delta, \beta_1, \dots, \beta_k \rangle_{n'}$$

Two schemas for MOVE rules:

$$\langle s_i s, s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f, \alpha_{i+1}, \dots, \alpha_k \rangle_0$$

$$\begin{cases} \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \delta, \alpha_{i+1}, \dots, \alpha_k \rangle_0 & \to \\ \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f\delta, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \end{cases}$$

#### One schema for INSERT rules:

Easy probabilities

$$\langle s, s_1, \dots, s_j, t, t_1, \dots, t_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_j, -f\gamma', \beta_1, \dots, \beta_k \rangle_n \rightarrow s, s_1, \dots, s_j :: \langle +f\gamma, \alpha_1, \dots, \alpha_j \rangle_n \quad \langle t, t_1, \dots, t_k \rangle :: \langle -f\gamma', \beta_1, \dots, \beta_k \rangle_{n'}$$

#### Three schemas for MRG rules:

$$\langle ss_i, s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f, \alpha_{i+1}, \dots, \alpha_k \rangle_1$$

$$\langle s_i s, s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \rightarrow \\ \langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f, \alpha_{i+1}, \dots, \alpha_k \rangle_0$$

$$\frac{\langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle \gamma, \alpha_1, \dots, \alpha_{i-1}, \delta, \alpha_{i+1}, \dots, \alpha_k \rangle_0 \rightarrow}{\langle s, s_1, \dots, s_i, \dots, s_k \rangle :: \langle +f\gamma, \alpha_1, \dots, \alpha_{i-1}, -f\delta, \alpha_{i+1}, \dots, \alpha_k \rangle_0}$$

# Subtlely different minimalist frameworks

Minimalist grammars with many choices of different bells and whistles can all be expressed with context-free derivational structure.

- Must keep an eye on finiteness of number of types (SMC or equivalent)!
- See Stabler (2011)

#### Some points of variation:

- adjunction
- head movement
- phases
- move as re-merge
- ...

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#### Some points of variation:

- adjunction
- head movement
- phases
- move as re-merge
- ...

Each variant of the formalism expresses a different hypothesis about the set of primitive grammatical operations. (We are looking for ways to tell these apart!)

- The "shapes" of the derivation trees are generally very similar from one variant to the next
- But variants will make different classifications of the derivational steps involved. according to which operation is being applied.

### Outline

- 15 Problem #1 with the naive parametrization

# Probabilities on MCFGs

Training question: What values of  $\lambda_1$ ,  $\lambda_2$ , etc. make the training corpus most likely?

# Problem #1 with the naive parametrization

# The 'often' Grammar: MGoften

pierre :: d who :: d - whwill := v = d tmarie :: d praise :: =d v  $\epsilon :: = t c$ often :: =v v  $\epsilon :: = t + wh c$ 

# Training data

90 pierre will praise marie pierre will often praise marie who pierre will praise who pierre will often praise

# Problem #1 with the naive parametrization

### The 'often' Grammar: MGoffen

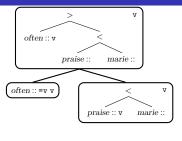
pierre :: d who :: d - whmarie :: d will :: =v =d t praise :: =d v  $\epsilon :: = t c$ often :: =v v  $\epsilon :: = t + wh c$ 

Easy probabilities

### Training data

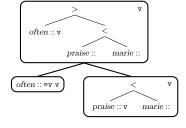
pierre will praise marie pierre will often praise marie who pierre will praise who pierre will often praise

# Generalizations missed by the naive parametrization

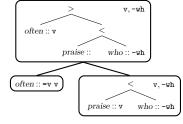


$$st :: \langle v \rangle_0 \rightarrow s :: \langle =v \ v \rangle_1 \quad t :: \langle v \rangle_0$$

# Generalizations missed by the naive parametrization



$$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1 \quad t :: \langle v \rangle_0$$



$$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v v \rangle_1 \langle t, u \rangle :: \langle v, -wh \rangle_0$$

# Problem #1 with the naive parametrization

### The 'often' Grammar: MGoffen

Easy probabilities

pierre :: d who :: d - whmarie :: d will :: =v =d t praise :: =d v  $\epsilon :: = t c$ often :: =v v  $\epsilon :: = t + wh c$ 

### Training data

pierre will praise marie pierre will often praise marie who pierre will praise who pierre will often praise

### The 'often' Grammar: MGoften

pierre :: d who :: d -wh marie :: d will :: =v =d t praise :: =d v  $\epsilon$  :: =t c often :: =v v  $\epsilon$  :: =t +wh c

### Training data

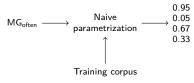
pierre will praise marie
pierre will often praise marie
who pierre will praise
who pierre will often praise

$$\frac{\mathsf{count} \Big( \langle \mathsf{v} \rangle_0 \, \to \, \langle \mathsf{=d} \; \mathsf{v} \rangle_1 \, \, \langle \mathsf{d} \rangle_1 \Big)}{\mathsf{count} \Big( \langle \mathsf{v} \rangle_0 \Big)} = \frac{95}{100}$$

$$\frac{\mathsf{count}\!\left(\langle\mathtt{v}\,\text{,-wh}\rangle_0\to\langle\mathtt{=d}\,\mathtt{v}\rangle_1\,\langle\mathtt{d}\,\mathtt{-wh}\rangle_1\right)}{\mathsf{count}\!\left(\langle\mathtt{v}\,\text{,-wh}\rangle_0\right)}=\frac{2}{3}$$

This training setup doesn't know which minimalist-grammar operations are being implemented by the various MCFG rules.

### Naive parametrization



### Outline

- Basy probabilities with context-free structure
- 14 Different frameworks
- Problem #1 with the naive parametrization
- 16 Problem #2 with the naive parametrization
- Solution: Faithfulness to MG operations

# A (slightly) more complicated grammar: MG<sub>shave</sub>

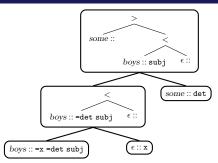
```
\epsilon :: = t c
                                   boys :: =x =det subj
\epsilon = t + wh c
                                   \epsilon :: x
will :: =v =subj t
                                   some :: det
shave " v
shave :: =obj v
                                   themselves :: =ant obj
boys :: subj
                                   \epsilon :: = \mathtt{subj} \; \mathtt{ant} \; - \mathtt{subj}
who :: subi -wh
                                   will :: =v +subi t
```

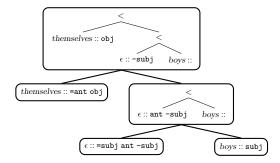
boys will shave boys will shave themselves who will shave who will shave themselves some boys will shave some boys will shave themselves

#### Some details:

Easy probabilities

- Subject is base-generated in SpecTP; no movement for Case
- Transitive and intransitive versions of shave
- some is a determiner that optionally combines with boys to make a subject
  - Dummy feature x to fill complement of boys so that some goes on the left
- themselves can appear in object position, via a movement theory of reflexives
  - A subj can be turned into an ant -subj
  - themselves combines with an ant to make an obj
  - will can attract its subject by move as well as merge





# Choice points in the MG-derived MCFG

### Question or not?

Easy probabilities

```
\langle c \rangle_0 \rightarrow \langle =t c \rangle_0 \langle t \rangle_0
 \langle c \rangle_0 \rightarrow \langle +wh c, -wh \rangle_0
```

### Antecedent lexical or complex?

```
\langle 	ext{ant -subj} 
angle_0 \quad 	o \quad \langle 	ext{=subj ant -subj} 
angle_1
                                                                                                         \langle \mathtt{subj} \rangle_0
\langle {
m ant -subj} 
angle_0 \ 	o \ \langle {
m =subj ant -subj} 
angle_1
                                                                                                         \langle \mathtt{subj} \rangle_1
```

### Non-wh subject merged and complex, merged and lexical, or moved?

```
\langle = \text{subj } t \rangle_0 \quad \langle \text{subj} \rangle_0
\rightarrow \langle = \text{subj t} \rangle_0 \langle \text{subj} \rangle_1
             \langle + \text{subj t}, - \text{subj} \rangle_0
```

### Wh-phrase same as moving subject or separated because of doubling?

$$\langle \mathtt{t}, \mathtt{-wh} \rangle_0 \rightarrow \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 \langle \mathtt{subj} \ \mathtt{-wh} \rangle_1$$
  
 $\langle \mathtt{t}, \mathtt{-wh} \rangle_0 \rightarrow \langle \mathtt{+subj} \ \mathtt{t}, \mathtt{-subj}, \mathtt{-wh} \rangle_0$ 

# Choice points in the IMG-derived MCFG

### Question or not?

Easy probabilities

```
\langle -c \rangle_0 \rightarrow \langle +t -c, -t \rangle_1
 \langle -c \rangle_0 \rightarrow \langle +wh -c, -wh \rangle_0
```

### Antecedent lexical or complex?

```
\langle + \text{subj-ant-subj}, - \text{subj} \rangle_0 \rightarrow \langle + \text{subj-ant-subj} \rangle_0
                                                                                                                  ⟨-subj⟩₀
\langle + \text{subj-ant-subj}, - \text{subj} \rangle_0 \rightarrow \langle + \text{subj-ant-subj} \rangle_0
                                                                                                                  \langle -subj \rangle_1
```

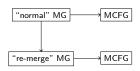
### Non-wh subject merged and complex, merged and lexical, or moved?

```
\langle + \text{subj} - t, - \text{subj} \rangle_0 \rightarrow \langle + \text{subj} - t \rangle_0 \langle - \text{subj} \rangle_0
 \langle + \mathtt{subj} - \mathtt{t}, - \mathtt{subj} 
angle_0 \ 	o \ \langle + \mathtt{subj} - \mathtt{t} 
angle_0 \ \langle - \mathtt{subj} 
angle_1
 \langle + \mathtt{subj} - \mathtt{t}, - \mathtt{subj} 
angle_0 \ 	o \ \langle + \mathtt{v} + \mathtt{subj} - \mathtt{t}, - \mathtt{v}, - \mathtt{subj} 
angle_1
```

### Wh-phrase same as moving subject or separated because of doubling?

$$\begin{array}{cccc} \langle -\mathtt{t}, -\mathtt{wh} \rangle_0 & \to & \langle +\mathtt{subj} -\mathtt{t}, -\mathtt{subj} -\mathtt{wh} \rangle_0 \\ \langle -\mathtt{t}, -\mathtt{wh} \rangle_0 & \to & \langle +\mathtt{subj} -\mathtt{t}, -\mathtt{subj}, -\mathtt{wh} \rangle_0 \end{array}$$

# Problem #2 with the naive parametrization

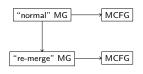


#### Language of both grammars

boys will shave boys will shave themselves who will shave who will shave themselves some boys will shave some boys will shave themselves

- 10 boys will shave
  - boys will shave themselves
- 3 who will shave
- 1 who will shave themselves
- 5 some boys will shave

# Problem #2 with the naive parametrization



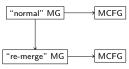
#### Language of both grammars

boys will shave boys will shave themselves who will shave who will shave themselves some boys will shave some boys will shave themselves

- 10 boys will shave
  - boys will shave themselves who will shave
- who will shave themselves
- 5 some boys will shave

e. merge and move distinct
boys will shave
some boys will shave
who will shave
boys will shave themselves
who will shave themselves

# Problem #2 with the naive parametrization



# Language of both grammars boys will shave themselves

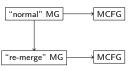
who will shave who will shave themselves some boys will shave some boys will shave themselves

boys will shave

- 10 boys will shave
  - boys will shave themselves who will shave
- who will shave themselves some boys will shave

MG <sub>shave</sub> , i.e	. merge and move distinct
0.47619	boys will shave
0.238095	some boys will shave
0.142857	who will shave
0.0952381	boys will shave themselves
0.047619	who will shave themselves

$IMG_{shave}$ ,	i.e. merge and move unified
0.47619	boys will shave
0.238095	some boys will shave
0.142857	who will shave
0.0952381	boys will shave themselves
0 047610	who will shave themselves



#### Language of both grammars

boys will shave themselves

who will shave
who will shave themselves
some boys will shave
some boys will shave themselves

bovs will shave

#### Training data

- boys will shaveboys will shave themselveswho will shave
- who will shave themselves some boys will shave

MG<sub>shave</sub>, i.e. merge and move distinct

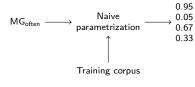
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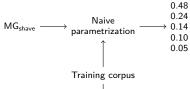
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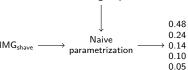
This treatment of probabilities doesn't know which derivational operations are being implemented by the various MCFG rules.

So the probabilities are unaffected by changes in set of primitive operations.

### Naive parametrization







- 17 Solution: Faithfulness to MG operations

Solution: Have a rule's probability be a function of (only) "what it does"

- merge or move
- what feature is being checked (either movement or selection)

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MCFG Rule	$\phi_{ ext{MERGE}}$	$\phi_{ t d}$	$\phi_{ t v}$	$\phi_{ t t}$	$\phi_{ ext{MOVE}}$	$\phi_{\mathtt{wh}}$
$st :: \langle c \rangle_0 \rightarrow s :: \langle =t c \rangle_1  t :: \langle t \rangle_0$	1	0	0	1	0	0
$ts :: \langle c \rangle_0 \rightarrow \langle s, t \rangle :: \langle +wh c, -wh \rangle_0$	0	0	0	0	1	1
$st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1  t :: \langle d \rangle_1$	1	1	0	0	0	0
$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1  t :: \langle v \rangle_0$	1	0	1	0	0	0
$\langle s, t \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -d v \rangle_1  t :: \langle d -wh \rangle_1$	1	1	0	0	0	0
$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v v \rangle_1  \langle t, u \rangle :: \langle v, -wh \rangle_0$	1	0	1	0	0	0

$$\begin{split} s(r) &= \exp(\lambda \cdot \phi(r)) \\ &= \exp(\lambda_{\text{MERGE}} \, \phi_{\text{MERGE}}(r) + \lambda_{\text{d}} \, \phi_{\text{d}}(r) + \lambda_{\text{v}} \, \phi_{\text{v}}(r) + \dots) \end{split}$$

Solution: Have a rule's probability be a function of (only) "what it does"

- merge or move
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MCFG Rule	$\phi_{ ext{MERGE}}$	$\phi_{ t d}$	$\phi_{\mathtt{v}}$	$\phi_{ t t}$	$\phi_{ ext{MOVE}}$	$\phi_{\mathtt{wh}}$
$st :: \langle c \rangle_0 \rightarrow s :: \langle =t c \rangle_1  t :: \langle t \rangle_0$	1	0	0	1	0	0
$ts :: \langle c \rangle_0 \rightarrow \langle s, t \rangle :: \langle +wh c, -wh \rangle_0$	0	0	0	0	1	1
$st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1  t :: \langle d \rangle_1$	1	1	0	0	0	0
$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1  t :: \langle v \rangle_0$	1	0	1	0	0	0
$\langle s, t \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -d v \rangle_1  t :: \langle d -wh \rangle_1$	1	1	0	0	0	0
$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v v \rangle_1  \langle t, u \rangle :: \langle v, -wh \rangle_0$	1	0	1	0	0	0

$$\begin{split} s(r) &= \exp(\lambda \cdot \phi(r)) \\ &= \exp(\lambda_{\text{MERGE}} \, \phi_{\text{MERGE}}(r) + \lambda_{\text{d}} \, \phi_{\text{d}}(r) + \lambda_{\text{v}} \, \phi_{\text{v}}(r) + \dots) \\ s(r_1) &= \exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) \end{split}$$

Solution: Have a rule's probability be a function of (only) "what it does"

- merge or move
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$st :: \langle c \rangle_0 \rightarrow s :: \langle =t c \rangle_1  t :: \langle t \rangle_0$	1	0	0	1	0	0
$ts :: \langle c \rangle_0 \rightarrow \langle s, t \rangle :: \langle +wh c, -wh \rangle_0$	0	0	0	0	1	1
$st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1  t :: \langle d \rangle_1$	1	1	0	0	0	0
$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1  t :: \langle v \rangle_0$	1	0	1	0	0	0
$\langle s, t \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -d v \rangle_1  t :: \langle d -wh \rangle_1$	1	1	0	0	0	0
$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v v \rangle_1  \langle t, u \rangle :: \langle v, -wh \rangle_0$	1	0	1	0	0	0

$$\begin{split} s(r) &= \exp(\lambda \cdot \phi(r)) \\ &= \exp(\lambda_{\text{MERGE}} \, \phi_{\text{MERGE}}(r) + \lambda_{\text{d}} \, \phi_{\text{d}}(r) + \lambda_{\text{v}} \, \phi_{\text{v}}(r) + \dots) \\ s(r_1) &= \exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) \\ s(r_2) &= \exp(\lambda_{\text{MOVE}} + \lambda_{\text{vh}}) \end{split}$$

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$st :: \langle c \rangle_0 \rightarrow s :: \langle =t c \rangle_1  t :: \langle t \rangle_0$	1	0	0	1	0	0
$ts :: \langle c \rangle_0 \rightarrow \langle s, t \rangle :: \langle +wh c, -wh \rangle_0$	0	0	0	0	1	1
$st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1  t :: \langle d \rangle_1$	1	1	0	0	0	0
$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1  t :: \langle v \rangle_0$	1	0	1	0	0	0
$\langle s, t \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -d v \rangle_1  t :: \langle d -wh \rangle_1$	1	1	0	0	0	0
$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle =v v \rangle_1  \langle t, u \rangle :: \langle v, -wh \rangle_0$	1	0	1	0	0	0

$$s(r) = \exp(\lambda \cdot \phi(r))$$

$$= \exp(\lambda_{\text{MERGE}} \phi_{\text{MERGE}}(r) + \lambda_{\text{d}} \phi_{\text{d}}(r) + \lambda_{\text{v}} \phi_{\text{v}}(r) + \dots)$$

$$s(r_1) = \exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}})$$

$$s(r_2) = \exp(\lambda_{\text{MOVE}} + \lambda_{\text{vh}})$$

$$s(r_3) = \exp(\lambda_{\text{MERGE}} + \lambda_{\text{d}})$$

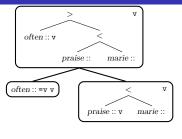
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- merge or move
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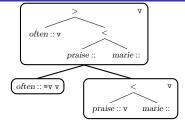
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$st :: \langle c \rangle_0 \rightarrow s :: \langle =t c \rangle_1  t :: \langle t \rangle_0$	1	0	0	1	0	0
$ts :: \langle c \rangle_0 \rightarrow \langle s, t \rangle :: \langle +wh c, -wh \rangle_0$	0	0	0	0	1	1
$st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1  t :: \langle d \rangle_1$	1	1	0	0	0	0
$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1  t :: \langle v \rangle_0$	1	0	1	0	0	0
$\langle s, t \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -d v \rangle_1  t :: \langle d -wh \rangle_1$	1	1	0	0	0	0
$\langle \mathit{st}, \mathit{u} \rangle  ::  \langle \mathtt{v}, -\mathtt{wh} \rangle_0 \ \rightarrow \ \mathit{s}  ::  \langle \mathtt{=v}  \mathtt{v} \rangle_1  \langle \mathit{t}, \mathit{u} \rangle  ::  \langle \mathtt{v}, -\mathtt{wh} \rangle_0$	1	0	1	0	0	0

$$\begin{split} s(r) &= \exp(\lambda \cdot \phi(r)) \\ &= \exp(\lambda_{\text{MERGE}} \, \phi_{\text{MERGE}}(r) + \lambda_{\text{d}} \, \phi_{\text{d}}(r) + \lambda_{\text{v}} \, \phi_{\text{v}}(r) + \dots) \\ s(r_1) &= \exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) \\ s(r_2) &= \exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}}) \\ s(r_3) &= \exp(\lambda_{\text{MERGE}} + \lambda_{\text{d}}) \\ s(r_5) &= \exp(\lambda_{\text{MERGE}} + \lambda_{\text{d}}) \end{split}$$

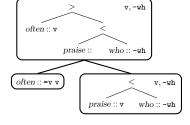
# Generalizations missed by the naive parametrization



$$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1 \quad t :: \langle v \rangle_0$$



$$st :: \langle v \rangle_0 \rightarrow s :: \langle =v v \rangle_1 \quad t :: \langle v \rangle_0$$



$$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v v \rangle_1 \langle t, u \rangle :: \langle v, -wh \rangle_0$$

### Comparison

### The old way:

Training question: What values of  $\lambda_1$ ,  $\lambda_2$ , etc. make the training corpus most likely?

### The new way:

Training question: What values of  $\lambda_{\rm MERGE}$ ,  $\lambda_{\rm MOVE}$ ,  $\lambda_{\rm d}$ , etc. make the training corpus most likely?

# Solution #1 with the smarter parametrization

#### Grammar

often :: =v v

Easy probabilities

pierre :: d who :: d - wh marie :: d will :: = v = d t praise :: = d v  $\epsilon :: = t c$ 

### Training data

90 pierre will praise marie
5 pierre will often praise marie
1 who pierre will praise

who pierre will often praise

Maximise likelihood via stochastic gradient ascent:

 $\epsilon :: = t + wh c$ 

$$P_{\lambda}(N \to \delta) = \frac{\exp(\lambda \cdot \phi(N \to \delta))}{\sum \exp(\lambda \cdot \phi(N \to \delta'))}$$

# Solution #1 with the smarter parametrization

#### Grammar

Easy probabilities

pierre :: d  $who \cdot \cdot d - wh$ marie · · d  $will \cdot \cdot = v = d t$ praise :: =d v  $\epsilon :: = t c$ often :: =v v  $\epsilon :: = t + wh c$ 

### Training data

pierre will praise marie pierre will often praise marie who pierre will praise

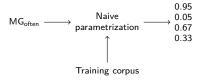
who pierre will often praise

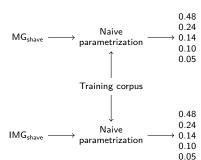
Maximise likelihood via stochastic gradient ascent:

$$P_{\lambda}(N \to \delta) = \frac{\exp(\lambda \cdot \phi(N \to \delta))}{\sum \exp(\lambda \cdot \phi(N \to \delta'))}$$

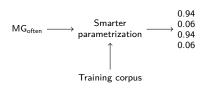
	naive	smarter
$st :: \langle v \rangle_0 \rightarrow s :: \langle =d v \rangle_1  t :: \langle d \rangle_1$	0.95	0.94
$st: \langle v \rangle_0 \rightarrow s:: \langle =v \ v \rangle_1  t:: \langle v \rangle_0$	0.05	0.06
$\langle s,t \rangle :: \langle \mathtt{v}, \mathtt{-wh} \rangle_0 \  o \ s :: \langle \mathtt{=d} \ \mathtt{v} \rangle_1 \ t :: \langle \mathtt{d} \ \mathtt{-wh} \rangle_1$	0.67	0.94
$\langle st, u \rangle :: \langle v, -wh \rangle_0 \rightarrow s :: \langle -v v \rangle_1 \langle t, u \rangle :: \langle v, -wh \rangle_0$	0.33	0.06

### Naive parametrization

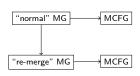




### **Smarter parametrization**



# Solution #2 with the smarter parametrization



#### Language of both grammars

boys will shave boys will shave themselves who will shave who will shave themselves some boys will shave some boys will shave themselves

- 10 boys will shave
- 2 boys will shave themselves
- 3 who will shave
- 1 who will shave themselves
- 5 some boys will shave

# "normal" MG MCFG "re-merge" MG MCFG

#### Language of both grammars

boys will shave

boys will shave themselves who will shave who will shave themselves some boys will shave some boys will shave themselves

#### Training data

- boys will shave 10 boys will shave themselves
- who will shave
- who will shave themselves
- some boys will shave

#### MG<sub>shave</sub>, i.e. merge and move distinct

0.35478	boys will shave
0.35478	some boys will shave
0.14801	who will shave
0.05022	boys will shave themselves
0.05022	some boys will shave themselves
0.04199	who will shave themselves

# "normal" MG MCFG "re-merge" MG MCFG

0.35478

#### Language of both grammars

boys will shave

boys will shave themselves who will shave who will shave themselves some boys will shave some boys will shave themselves

#### Training data

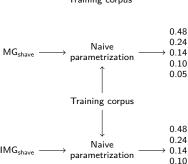
- boys will shave 10 boys will shave themselves
- who will shave
- who will shave themselves some boys will shave

#### MG<sub>shave</sub>, i.e. merge and move distinct have will shave

0.55110	boys will shave
0.35478	some boys will shave
0.14801	who will shave
0.05022	boys will shave themselves
0.05022	some boys will shave themselves
0.04199	who will shave themselves

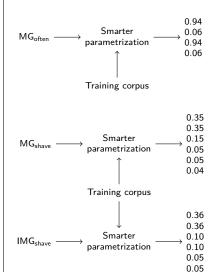
#### IMGshave, i.e. merge and move unified

0.35721	boys will shave
0.35721	some boys will shave
0.095	who will shave
0.095	who will shave themselves
0.04779	boys will shave themselves
0.04779	some boys will shave themselves



0.05

## **Smarter parametrization**



#### Question or not?

Easy probabilities

$$\begin{array}{cccc} \langle \mathtt{c} \rangle_0 & \to & \langle \mathtt{=t} \ \mathtt{c} \rangle_0 & \langle \mathtt{t} \rangle_0 \\ \langle \mathtt{c} \rangle_0 & \to & \langle \mathtt{+wh} \ \mathtt{c}, \mathtt{-wh} \rangle_0 \end{array}$$

#### Antecedent lexical or complex?

```
\begin{array}{lll} \langle \text{ant-subj} \rangle_0 & \rightarrow & \langle = \text{subj ant-subj} \rangle_1 & \langle \text{subj} \rangle_0 \\ \langle \text{ant-subj} \rangle_0 & \rightarrow & \langle = \text{subj ant-subj} \rangle_1 & \langle \text{subj} \rangle_1 \end{array}
```

### Non-wh subject merged and complex, merged and lexical, or moved?

```
\begin{array}{lll} \langle \mathtt{t} \rangle_0 & \to & \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 & \langle \mathtt{subj} \rangle_0 \\ \langle \mathtt{t} \rangle_0 & \to & \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 & \langle \mathtt{subj} \rangle_1 \\ \langle \mathtt{t} \rangle_0 & \to & \langle \mathtt{+subj} \ \mathtt{t}, \mathtt{-subj} \rangle_0 \end{array}
```

$$\langle \mathtt{t}, \mathtt{-wh} \rangle_0 \rightarrow \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 \langle \mathtt{subj} \ \mathtt{-wh} \rangle_1$$
  
 $\langle \mathtt{t}, \mathtt{-wh} \rangle_0 \rightarrow \langle \mathtt{+subj} \ \mathtt{t}, \mathtt{-subj}, \mathtt{-wh} \rangle_0$ 

## Choice points in the MG-derived MCFG

### Question or not?

$$\begin{array}{cccc} \langle \mathtt{c} \rangle_0 & \rightarrow & \langle \mathtt{=t} \ \mathtt{c} \rangle_0 & \langle \mathtt{t} \rangle_0 & \exp(\lambda_{\mathrm{MERGE}} + \lambda_\mathtt{t}) \\ \langle \mathtt{c} \rangle_0 & \rightarrow & \langle \mathtt{+wh} \ \mathtt{c}, \mathtt{-wh} \rangle_0 & \exp(\lambda_{\mathrm{MOVE}} + \lambda_\mathtt{wh}) \end{array}$$

#### Antecedent lexical or complex?

```
\begin{array}{lll} \langle \text{ant-subj} \rangle_0 & \rightarrow & \langle = \text{subj ant-subj} \rangle_1 & \langle \text{subj} \rangle_0 & \exp(\lambda_{\text{MERGE}} + \lambda_{\text{subj}}) \\ \langle \text{ant-subj} \rangle_0 & \rightarrow & \langle = \text{subj ant-subj} \rangle_1 & \langle \text{subj} \rangle_1 & \exp(\lambda_{\text{MERGE}} + \lambda_{\text{subj}}) \end{array}
```

### Non-wh subject merged and complex, merged and lexical, or moved?

```
\begin{array}{lll} \langle \mathtt{t} \rangle_0 & \to & \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 & \langle \mathtt{subj} \rangle_0 & \exp(\lambda_{\mathtt{MERGE}} + \lambda_{\mathtt{subj}}) \\ \langle \mathtt{t} \rangle_0 & \to & \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 & \langle \mathtt{subj} \rangle_1 & \exp(\lambda_{\mathtt{MERGE}} + \lambda_{\mathtt{subj}}) \\ \langle \mathtt{t} \rangle_0 & \to & \langle \mathtt{+subj} \ \mathtt{t}, \mathtt{-subj} \rangle_0 & \exp(\lambda_{\mathtt{MOVE}} + \lambda_{\mathtt{subj}}) \end{array}
```

$$\begin{array}{lll} \langle \mathtt{t}, -\mathtt{wh} \rangle_0 & \to & \langle \mathtt{=subj} \ \mathtt{t} \rangle_0 & \langle \mathtt{subj} \ -\mathtt{wh} \rangle_1 & & \exp(\lambda_{\mathrm{MERGE}} + \lambda_{\mathtt{subj}}) \\ \langle \mathtt{t}, -\mathtt{wh} \rangle_0 & \to & \langle \mathtt{+subj} \ \mathtt{t}, -\mathtt{subj}, -\mathtt{wh} \rangle_0 & & \exp(\lambda_{\mathrm{MOVE}} + \lambda_{\mathtt{subj}}) \end{array}$$

# Choice points in the IMG-derived MCFG

#### Question or not?

$$\begin{array}{cccc} \langle -c \rangle_0 & \rightarrow & \langle +t \ -c, -t \rangle_1 \\ \langle -c \rangle_0 & \rightarrow & \langle +wh \ -c, -wh \rangle_0 \end{array}$$

### Antecedent lexical or complex?

```
\begin{array}{lll} \langle + \text{subj} - \text{ant} - \text{subj}, - \text{subj} \rangle_0 & \rightarrow & \langle + \text{subj} - \text{ant} - \text{subj} \rangle_0 & \langle - \text{subj} \rangle_0 \\ \langle + \text{subj} - \text{ant} - \text{subj}, - \text{subj} \rangle_0 & \rightarrow & \langle + \text{subj} - \text{ant} - \text{subj} \rangle_0 & \langle - \text{subj} \rangle_1 \end{array}
```

### Non-wh subject merged and complex, merged and lexical, or moved?

```
\begin{array}{lll} \langle + \text{subj} - \text{t}, - \text{subj} \rangle_0 & \rightarrow & \langle + \text{subj} - \text{t} \rangle_0 & \langle - \text{subj} \rangle_0 \\ \langle + \text{subj} - \text{t}, - \text{subj} \rangle_0 & \rightarrow & \langle + \text{subj} - \text{t} \rangle_0 & \langle - \text{subj} \rangle_1 \\ \langle + \text{subj} - \text{t}, - \text{subj} \rangle_0 & \rightarrow & \langle + \text{v} + \text{subj} - \text{t}, - \text{v}, - \text{subj} \rangle_1 \end{array}
```

$$\begin{array}{cccc} \langle -\mathtt{t}, -\mathtt{w} \mathtt{h} \rangle_0 & \to & \langle +\mathtt{subj} -\mathtt{t}, -\mathtt{subj} -\mathtt{w} \mathtt{h} \rangle_0 \\ \langle -\mathtt{t}, -\mathtt{w} \mathtt{h} \rangle_0 & \to & \langle +\mathtt{subj} -\mathtt{t}, -\mathtt{subj}, -\mathtt{w} \mathtt{h} \rangle_0 \end{array}$$

## Choice points in the IMG-derived MCFG

### Question or not?

Easy probabilities

$$\begin{array}{cccc} \langle -c \rangle_0 & \rightarrow & \langle +t-c, -t \rangle_1 & & exp(\lambda_{\rm MRG} + \lambda_t) \\ \langle -c \rangle_0 & \rightarrow & \langle +\text{wh} -c, -\text{wh} \rangle_0 & & exp(\lambda_{\rm MRG} + \lambda_{\text{wh}}) \end{array}$$

#### Antecedent lexical or complex?

```
\langle + \text{subj-ant-subj}, - \text{subj} \rangle_0 \rightarrow \langle + \text{subj-ant-subj} \rangle_0
                                                                                                                             \langle -subj \rangle_0
                                                                                                                                                        \exp(\lambda_{\text{INSERT}})
                                                                                                                                                        \exp(\lambda_{\text{INSERT}})
\langle + \mathtt{subj-ant-subj}, -\mathtt{subj} 
angle_0 \quad 	o \quad
                                                                        \langle +subj -ant -subj \rangle_0
                                                                                                                             \langle -subj \rangle_1
```

### Non-wh subject merged and complex, merged and lexical, or moved?

```
\langle +\text{subj} - t, -\text{subj} \rangle_0 \rightarrow \langle +\text{subj} - t \rangle_0 \langle -\text{subj} \rangle_0
                                                                                                                                                                                             \exp(\lambda_{\text{INSERT}})
 \langle + \operatorname{subj} - \operatorname{t}, - \operatorname{subj} \rangle_0 \rightarrow \langle + \operatorname{subj} - \operatorname{t} \rangle_0 \langle - \operatorname{subj} \rangle_1
                                                                                                                                                                                             \exp(\lambda_{\text{INSERT}})
 \langle + \operatorname{subj} - \operatorname{t}, - \operatorname{subj} \rangle_0 \quad 	o \quad \langle + \operatorname{v} + \operatorname{subj} - \operatorname{t}, -\operatorname{v}, - \operatorname{subj} \rangle_1
                                                                                                                                                                                             \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})
```

$$\begin{array}{lll} \langle -\mathtt{t}, -\mathtt{wh} \rangle_0 & \to & \langle +\mathtt{subj} - \mathtt{t}, -\mathtt{subj} - \mathtt{wh} \rangle_0 & & \exp(\lambda_{\text{MRG}} + \lambda_{\text{subj}}) \\ \langle -\mathtt{t}, -\mathtt{wh} \rangle_0 & \to & \langle +\mathtt{subj} - \mathtt{t}, -\mathtt{subj}, -\mathtt{wh} \rangle_0 & & \exp(\lambda_{\text{MRG}} + \lambda_{\text{subj}}) \end{array}$$

# Learned weights on the MG

 $\lambda_{\rm t}=0.094350$  $\exp(\lambda_{\rm t}) = 1.0989$  $\lambda_{\text{subj}} = -5.734063$  $\exp(\lambda_{\rm v})=0.0032$  $\lambda_{\text{wh}} = -0.094350$  $\exp(\lambda_{\mathrm{wh}}) = 0.9100$  $\lambda_{\text{MERGE}} = 0.629109$  $\exp(\lambda_{\text{MERGE}}) = 1.8759$  $\lambda_{\text{MOVE}} = -0.629109$  $\exp(\lambda_{\text{MOVE}}) = 0.5331$  Easy probabilities

$$P(\text{antecedent is lexical}) = 0.5$$
 
$$\lambda_t = 0.094350 \qquad \exp(\lambda_t) = 1.0989 \qquad P(\text{antecedent is non-lexical}) = 0.5$$
 
$$\lambda_{\text{Bubj}} = -5.734063 \qquad \exp(\lambda_v) = 0.0032$$
 
$$\lambda_{\text{uh}} = -0.094350 \qquad \exp(\lambda_{\text{wh}}) = 0.9100$$
 
$$\lambda_{\text{MERGE}} = 0.629109 \qquad \exp(\lambda_{\text{MERGE}}) = 1.8759$$
 
$$P(\text{wh-phrase reflexivized}) = \frac{\exp(\lambda_{\text{MOVE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.2213$$
 
$$\lambda_{\text{MOVE}} = -0.629109 \qquad \exp(\lambda_{\text{MOVE}}) = 0.5331$$
 
$$P(\text{wh-phrase non-reflexivized}) = \frac{\exp(\lambda_{\text{MERGE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.7787$$

$$\begin{split} P(\text{question}) &= \frac{\exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}})}{\exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}})} = 0.1905 \\ P(\text{non-question}) &= \frac{\exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}})}{\exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}})} = 0.8095 \end{split}$$

$$\begin{split} P(\text{non-wh subject merged and complex}) &= \frac{\exp(\lambda_{\text{MERGE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.4378 \\ P(\text{non-wh subject merged and lexical}) &= \frac{\exp(\lambda_{\text{MERGE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.4378 \\ P(\text{non-wh subject moved}) &= \frac{\exp(\lambda_{\text{MOVE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.1244 \end{split}$$

Different frameworks Solution: Faithfulness to MG operations

# Learned weights on the MG

 $\lambda_{\text{MOVE}} = -0.629109$ 

## P(antecedent is lexical) = 0.5

$$\lambda_{\rm t} = 0.094350$$
  $\exp(\lambda_{\rm t}) = 1.0989$   $P({\rm antecedent~is~non-lexical}) = 0.5$ 

$$\lambda_{\text{subj}} = -5.734063 \qquad \text{exp}(\lambda_{\text{v}}) = 0.0032$$

 $\lambda_{\rm wh} = -0.094350$  $\exp(\lambda_{\rm wh}) = 0.9100$  $P(\text{wh-phrase reflexivized}) = \frac{\exp(\lambda_{\text{MOVE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.2213$  $\lambda_{\text{MERGE}} = 0.629109$  $\exp(\lambda_{ ext{MERGE}}) = 1.8759$ 

 $\exp(\lambda_{\text{MOVE}}) = 0.5331$ 

$$\exp(\lambda_{ ext{MOVE}}) = 0.5331$$
  $P( ext{wh-phrase non-reflexivized}) = \frac{\exp(\lambda_{ ext{MERGE}})}{\exp(\lambda_{ ext{MERGE}}) + \exp(\lambda_{ ext{MOVE}})} = 0.7787$ 

$$\begin{split} \textit{P}(\text{question}) &= \frac{\exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}})}{\exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}})} = 0.1905 \\ \textit{P}(\text{non-question}) &= \frac{\exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}})}{\exp(\lambda_{\text{MERGE}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MOVE}} + \lambda_{\text{wh}})} = 0.8095 \end{split}$$

$$P(\text{non-wh subject merged and complex}) = \frac{\exp(\lambda_{\text{MERGE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.4378$$

$$P(\text{non-wh subject merged and lexical}) = \frac{\exp(\lambda_{\text{MERGE}})}{\exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MERGE}}) + \exp(\lambda_{\text{MOVE}})} = 0.4378$$

$$P( ext{non-wh subject moved}) = rac{\exp(\lambda_{ ext{MOVE}})}{\exp(\lambda_{ ext{MERGE}}) + \exp(\lambda_{ ext{MERGE}}) + \exp(\lambda_{ ext{MOVE}})} = 0.1244$$
  $P( ext{who will shave}) = 0.1905 imes 0.7787 = 0.148$ 

# Learned weights on the IMG

$$\begin{array}{lll} \lambda_t = 0.723549 & \exp(\lambda_t) = 2.0617 \\ \lambda_{\psi} = 0.440585 & \exp(\lambda_{\psi}) = 1.5536 \\ \lambda_{wh} = -0.723459 & \exp(\lambda_{wh}) = 0.4850 \\ \lambda_{INSERT} = 0.440585 & \exp(\lambda_{INSERT}) = 1.5536 \\ \lambda_{MRG} = -0.440585 & \exp(\lambda_{MRG}) = 0.6437 \end{array}$$

# Learned weights on the IMG

$$\begin{array}{lll} \lambda_{t}=0.723549 & \exp(\lambda_{t})=2.0617 & P(\text{antecedent is lexical})=0.5 \\ \lambda_{v}=0.440585 & \exp(\lambda_{v})=1.5536 & P(\text{antecedent is non-lexical})=0.5 \\ \lambda_{vh}=-0.723459 & \exp(\lambda_{vh})=0.4850 \\ \lambda_{\text{INSERT}}=0.440585 & \exp(\lambda_{\text{INSERT}})=1.5536 & P(\text{wh-phrase reflexivized})=0.5 \\ \lambda_{\text{MRG}}=-0.440585 & \exp(\lambda_{\text{MRG}})=0.6437 & P(\text{wh-phrase non-reflexivized})=0.5 \end{array}$$

$$\begin{split} P(\text{question}) &= \frac{\exp(\lambda_{\text{MRG}} + \lambda_{\text{wh}})}{\exp(\lambda_{\text{MRG}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{wh}})} = \frac{\exp(\lambda_{\text{wh}})}{\exp(\lambda_{\text{t}}) + \exp(\lambda_{\text{wh}})} = 0.1905 \\ P(\text{non-question}) &= \frac{\exp(\lambda_{\text{MRG}} + \lambda_{\text{t}})}{\exp(\lambda_{\text{MRG}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{wh}})} = \frac{\exp(\lambda_{\text{t}})}{\exp(\lambda_{\text{t}}) + \exp(\lambda_{\text{wh}})} = 0.8095 \end{split}$$

$$P(\text{non-wh subject merged and lexical}) = \frac{\exp(\lambda_{\text{INSERT}})}{\exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})} = 0.4412$$

$$P(\text{non-wh subject merged and complex}) = \frac{\exp(\lambda_{\text{INSERT}})}{\exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})} = 0.4412$$

$$P(\text{non-wh subject moved}) = \frac{\exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})}{\exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})} = 0.1176$$

# Learned weights on the IMG

$$\begin{array}{lll} \lambda_{\rm t} = 0.723549 & \exp(\lambda_{\rm t}) = 2.0617 & P({\rm antecedent~is~lexical}) = 0.5 \\ \lambda_{\rm v} = 0.440585 & \exp(\lambda_{\rm v}) = 1.5536 & P({\rm antecedent~is~non-lexical}) = 0.5 \\ \lambda_{\rm uh} = -0.723459 & \exp(\lambda_{\rm uh}) = 0.4850 \\ \lambda_{\rm INSERT} = 0.440585 & \exp(\lambda_{\rm INSERT}) = 1.5536 & P({\rm wh-phrase~reflexivized}) = 0.5 \\ \lambda_{\rm MRG} = -0.440585 & \exp(\lambda_{\rm MRG}) = 0.6437 & P({\rm wh-phrase~non-reflexivized}) = 0.5 \end{array}$$

$$\begin{split} P(\text{question}) &= \frac{\exp(\lambda_{\text{MRG}} + \lambda_{\text{vh}})}{\exp(\lambda_{\text{MRG}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{vh}})} = \frac{\exp(\lambda_{\text{vh}})}{\exp(\lambda_{\text{t}}) + \exp(\lambda_{\text{vh}})} = 0.1905 \\ P(\text{non-question}) &= \frac{\exp(\lambda_{\text{MRG}} + \lambda_{\text{t}})}{\exp(\lambda_{\text{MRG}} + \lambda_{\text{t}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{vh}})} = \frac{\exp(\lambda_{\text{t}})}{\exp(\lambda_{\text{t}}) + \exp(\lambda_{\text{vh}})} = 0.8095 \end{split}$$

$$P(\text{non-wh subject merged and lexical}) = \frac{\exp(\lambda_{\text{INSERT}})}{\exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})} = 0.4412$$

$$P(\text{non-wh subject merged and complex}) = \frac{\exp(\lambda_{\text{INSERT}})}{\exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})} = 0.4412$$

$$P(\text{non-wh subject moved}) = \frac{\exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})}{\exp(\lambda_{\text{INSERT}}) + \exp(\lambda_{\text{MRG}} + \lambda_{\text{v}})} = 0.1176$$

$$P(\text{who will shave}) = 0.5 \times 0.1905 = 0.095$$
 
$$P(\text{boys will shave themselves}) = 0.5 \times 0.8095 \times 0.1176 = 0.048$$

Grammar: MG<sub>shave</sub>

_			
	$MG_{shave},$ i.e. merge and move distinct		
	0.35478	boys will shave	
	0.35478	some boys will shave	
	0.14801	who will shave	
	0.05022	boys will shave themselves	
	0.05022	some boys will shave themselves	
	0.04199	who will shave themselves	

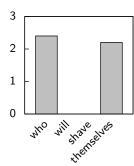
140

## Surprisal predictions

Grammar: MG<sub>shave</sub>

MG <sub>shave</sub> , i.e. merge and move distinct	
0.35478	boys will shave
0.35478	some boys will shave
0.14801	who will shave
0.05022	boys will shave themselves
0.05022	some boys will shave themselves
0.04199	who will shave themselves

surprisal at 'who' = 
$$-\log P(W_1 = \text{who})$$
  
=  $-\log(0.15 + 0.04)$   
=  $-\log 0.19$   
=  $2.4$   
surprisal at 'themselves' =  $-\log P(W_4 = \text{themselves} \mid W_1 = \text{who}, \dots)$   
=  $-\log \frac{0.04}{0.15 + 0.04}$   
=  $-\log 0.21$   
=  $2.2$ 



## Surprisal predictions

Grammar: IMG<sub>shave</sub>

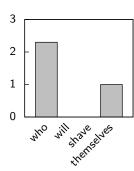
IMG <sub>sha</sub>	ve, i.e. merge and move unified
0.35721	boys will shave
0.35721	some boys will shave
0.095	who will shave
0.095	who will shave themselves
0.04779	boys will shave themselves
0.04779	some boys will shave themselves

# Surprisal predictions

Grammar: IMG<sub>shave</sub>

IMG <sub>shave</sub> , i.e. merge and move unified	
0.35721	boys will shave
0.35721	some boys will shave
0.095	who will shave
0.095	who will shave themselves
0.04779	boys will shave themselves
0.04779	some boys will shave themselves

surprisal at 'who' = 
$$-\log P(W_1 = \text{who})$$
  
=  $-\log(0.10 + 0.10)$   
=  $-\log 0.2$   
=  $2.3$   
surprisal at 'themselves' =  $-\log P(W_4 = \text{themselves} \mid W_1 = \text{who}, \dots)$   
=  $-\log \frac{0.10}{0.10 + 0.10}$   
=  $-\log 0.5$   
=  $1$ 



Part 1: Grammars and cognitive hypotheses

What is a grammar?

What can grammars do?
Concrete illustration of a target: Surprisal

Parts 2–4: Assembling the pieces

Minimalist Grammars (MGs)
MGs and MCEGs

Probabilities on MGs

Part 5: Learning and wrap-up

Something slightly different: Learning model Recap and open questions

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